a linear inch per second. At what rate per second is the angular velocity of the sphere changing the instant the diameter becomes $p=10$ inches? What is the diameter of the sphere when the rate of disappearance of matter is midway between minimum and maximum? When is the angular velocity a maximum? How does this maximum angular velocity compare with the original angular velocity? What is the diameter of the sphere when the paracentric force is (1) a maximum, and (2) a minimum?

No solution of this problem has been received.

140. Proposed by J. F. LAWRENCE, A. B., St. Louis. Mo.

A long row of particles, each mass m, is placed on a smooth horizontal table. Each is connected with the two adjacent ones by similar light elastic strings of natural length l. They receive small longitudinal disturbances such that each of them proceeds to perform a harmonic oscillation. Prove that there will be two waves of vibration in opposite directions with the same velocity, viz, $l'\sqrt{\frac{E}{ml}}\frac{q}{\pi}\sin\frac{\pi}{q}$, when l' is the average distance between two successive particles, q the number of intervals between two particles in the same phase, and E the modulus of elasticity. [*Mathematical Tripos*, 1873.]

Solution by G. B. M ZERR. A.M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia. Pa.

Let $D=d/dt$, and the equation of motion is $y_{k+1}-2y_k+y_{k-1}=\frac{D^2}{c^2}y_k$ where $c^2=E/(ml)$. To solve this equation of differences, treat D as constant and put $y_k=Cx^k$ where C and x are two constants. Substituting and reducing, $x-2+1/x=(D/c)^2$.

$$\therefore \sqrt{x}-1/\sqrt{x}=\pm D/c. \quad \sqrt{x}+1/\sqrt{x}=\pm\sqrt{\{4+(D/c)^2\}}=\pm 2\sqrt{1+\left(\frac{D}{2c}\right)^2}$$

$$\therefore \sqrt{x}=\sqrt{1+\left(\frac{D}{2c}\right)^2}-\frac{D}{2c}=E.$$

$$\therefore y_k=E^{2k}f(t)+E^{-2k}F(t).$$

Let us express $f(t)$ and $F(t)$ in a series whose general term is $A\cos(2c\sin\theta t+\omega)$.

The operator E under the radical contains only even powers of D and we can write $-(2c\sin\theta)^2$ for D^2.

$$\therefore E\cos(2c\sin\theta t+\omega)=\cos(2c\sin\theta t+\omega-\theta).$$

$$\therefore E^{2k}\cos(2c\sin\theta t+\omega)=\cos(2c\sin\theta t+\omega-2k\theta).$$

$$E^{-2k}\cos(2c\sin\theta t+\omega)=\cos(2c\sin\theta t+\omega+2k\theta).$$

$$\therefore y_k=\Sigma A\cos(2c\sin\theta t+\omega-2k\theta)+\Sigma B\cos(2c\sin\theta t+\omega+2k\theta).$$

If we substitute in any one term of the first series $k+1$ for k and $t+T$ for t, where $T=\frac{\theta}{c\sin\theta}$, the term is unaltered..

$\therefore$ Any one term of the first series represents a wave which travels the space between one particle and the next in time T. In the same way the corresponding term of the second series represents a wave which travels in the opposite direction with the same velocity.

Now v=velocity$=l'/T=l'c\sin\theta/\theta$; but $\theta=\pi/q$ and $c=\sqrt{\frac{E}{ml}}$.

$\therefore v=l'\sqrt{\frac{E}{ml}}\frac{q}{\pi}\sin\frac{\pi}{q}$.

141. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud. England.

A simple pendulum hangs from a bicycle moving in a straight line. What deflection is produced by putting on the brake so as to exert on the machine a force equal to the nth of its weight?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College. Philadelphia, Pa.

Let v=velocity of retardation, W=weight.

Then $v=\frac{Wg}{nW}t=\frac{g}{n}t$. Let $t=1$. $\therefore v=g/n$.

If θ=angle of deflection of the pendulum, and l=its length, then $v^2=g^2/n^2=2gl(1-\cos\theta)$.

$\therefore \cos\theta=\frac{2ln^2-g}{2ln^2}$ or $\theta=\cos^{-1}\left(\frac{2ln^2-g}{2ln^2}\right)$.

142. Proposed by GEORGE R. DEAN, B. Sc., Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo,

An infinite mass of liquid is bounded by the plane zx, on which are small corrugations given by $y=\phi(x)$. The velocity of the liquid at an infinite distance from the plane is parallel to x and equal to V. Prove that the velocity potential is $V_x+\frac{V}{\pi}\int_{-\infty}^{\infty}\frac{(x-\lambda)\phi(\lambda)d\lambda}{y^2+(x-\lambda)^2}$. [Bassett's *Hydrodynamics*.]

Solution by G. B. M. ZERR, A.M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let f=velocity potential.

Then $\frac{d^2f}{dx^2}+\frac{d^2f}{dy^2}=\frac{d^2f}{dx^2}\left(1-\frac{1}{[\varphi'(x)]^2}\right)=0$.

$\therefore \frac{df}{dx}$=constant$=v$ when $y=\infty$. $\therefore f-C=v\int dx=v\varphi_1(x)$.

When $y=0$, $f=\varphi_1(x)v$; when $y=\infty$, $\varphi(x)=x$.

$\therefore C=vx$. $\therefore f=vx+v\varphi_1(x)$.

By Fourier's series, $\varphi_1(x)=\frac{1}{\pi}\int_{-\infty}^{\infty}\varphi(\lambda)d\lambda\int_0^{\infty}\sin\beta(x-\lambda)d\beta$.